Lesson 7: Thermal and Mechanical Element Math Models in Control Systems

ET 438a
Automatic Control Systems Technology

Learning Objectives

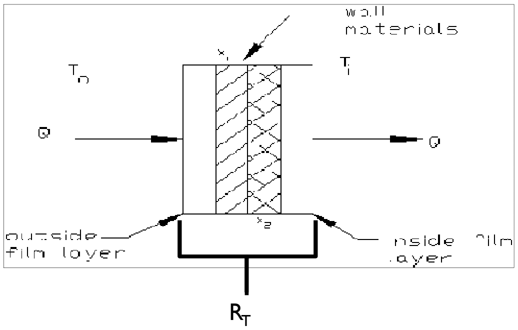
After this presentation you will be able to:

- Explain how heat flows from conduction and convection in a thermal system.
- Compute thermal resistance and determine heat flow.
- Determine thermal capacitance.
- Determine the mechanical resistance, capacitance and dead-time delay in a mechanical system.

Thermal Elements In Control Systems

Thermal Conduction and Convection

Thermal Resistance - opposition to heat flow. Heat flows from high temperature to low temperature.



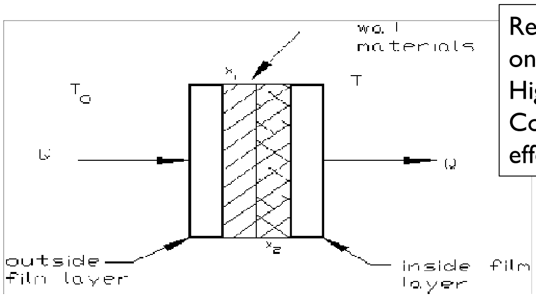
$$Q = \frac{T_o - T_i}{R_T}$$ Watts

R_u = unit thermal resistance

$$R_T = \frac{R_u}{A} \quad K^\circ / W$$
$$Q = \frac{(T_0 - T_i)\cdot A}{R_u} \quad W$$

Heat transfer by convection & conduction through R_T

Thermal Conduction and Convection

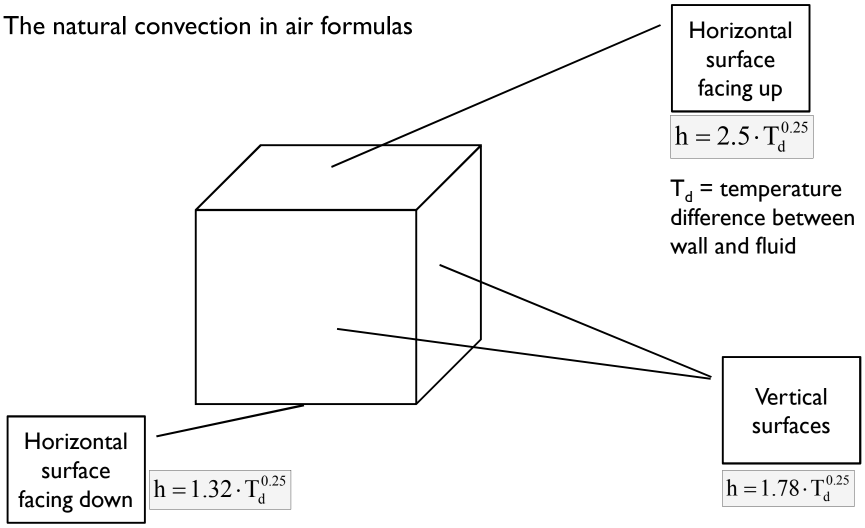


Resistance of film layers depends on type of fluid, velocity of fluid. High velocity flow makes thin film. Convection cooling.,“wind chill“ effect.

Film conductance $h = \frac{1}{R_{film}}$ is called film coefficient

Thermal Conduction and Convection

The natural convection in air formulas



Horizontal surface facing up

$h = 2.5 \cdot T_d^{0.25}$

T_d = temperature difference between wall and fluid

Vertical surfaces

$h = 1.78 \cdot T_d^{0.25}$

Horizontal surface facing down

$h = 1.32 \cdot T_d^{0.25}$

Thermal Conduction and Convection

Natural and Forced Convection

Natural convection in still water $h = 2.26 \cdot (T_w + 34.3) \cdot T_d^{0.5}$

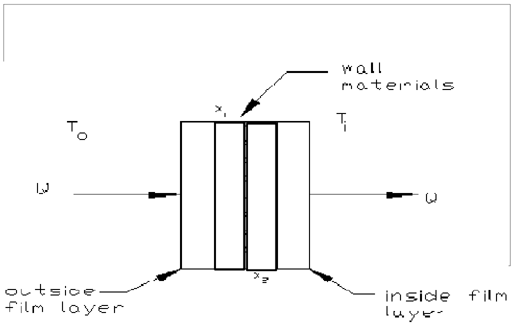
Where: T_w = water temperature

Forced convection-air against smooth surfaces and inside pipes.

$$
\begin{array}{ll}
v_{air} \le 4.6\ \mathrm{m/s} & h = 4.54 + 4.1 \cdot v_{air} \\
v_{air} > 4.6\ \mathrm{m/s} & h = 7.75 v_{air}^{0.75}
\end{array}
$$

Where: v_{air} = velocity of air flow.

Thermal Conduction



Thermal resistance of inner solid layers given by

$$R_{Ui} = \frac{x_i}{K_i} \quad \text{K-m}^2\text{/W}$$

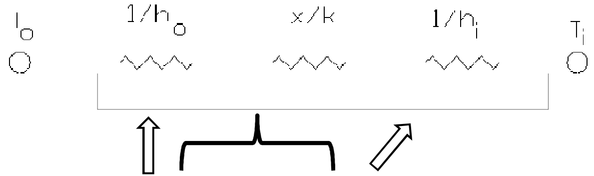
Where: x_i = thickness of i-th material
K_i = thermal conductivity of material

(See appendix in text for values)

Unit resistance of a composite wall is the sum of inside and outside film resistances and the resistances comprising the wall.

Thermal Conduction

Series resistance analogy for thermal conduction through a composite material. Temperature analogous to voltages.

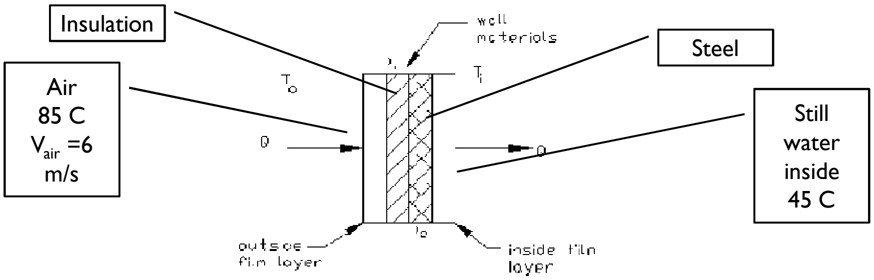


$$R_U = \frac{1}{h_o} + \frac{x_1}{K_1} + \frac{x_1}{K_1} + \ldots \frac{x_n}{K_n} + \frac{1}{h_i} \quad \text{K - m}^2\text{ / W}$$

$$R_T = \left(\frac{1}{A}\right)\cdot\left[\frac{1}{h_o} + \frac{x_1}{K_1} + \frac{x_1}{K_1} + \ldots \frac{x_n}{K_n} + \frac{1}{h_i}\right] \quad \text{K / W}$$

Heat Flow Example

Example 7-1: A wall section shown below has 2 layers: steel 1 cm thick, and insulation 2 cm thick. Still Water at 45 C is inside the wall. The temperature difference between the water and the inner wall is estimated to be 10 C. The outer wall is surrounded by air at a temperature of 85 C that has a velocity of 6 m/s. The wall dimensions are 2 m x 3 m. Determine the unit thermal resistance, the total thermal resistance, the total heat flow, the direction of heat flow.



Insulation

Air
85 C
V_{air} =6
m/s

Steel

Still
water
inside
45 C

Heat Flow Example Solution (1)

Find K_1 and K_2 from the Appendix A of the textbook

Steel $K_1 = 45$ W/m-K

Insulation $K_2 = 0.036$ W/m-K

$X_1 = 0.01$ m = 1 cm

$X_2 = 0.02$ m = 2cm

$A = 2m \times 3m = 6m^2$

Inside film coefficient is for still water

$T_w = 45$ C

$T_d = 10$ C

$h_i = 2.26(T_w + 34.3)T_d^{0.5}$

$h_i = 2.26(45 + 34.3)(10)^{0.5}$

$h_i = 2.26(79.3)(3.162)$ W/m²K

$h_i = 566.7$ W/m²-K

Heat Flow Example Solution (2)

Outside film coefficient is for force air convection

$$V_{air} = 6\ m/s > 4.6\ m/s$$

use $h_o = 7.76 V_{air}^{0.75}$

$$h_o = 7.76(6\ m/s)^{0.75}$$

$$h_o = 29.75\ W/m^2\text{-}K$$

$$R_u = \frac{1}{h_o} + \frac{X_1}{K_1} + \frac{X_2}{K_2} + \frac{1}{h_i} \qquad R_u = \frac{1}{29.75\ W/m^2\text{-}K} + \frac{0.01\ m}{45\ W/m\text{-}K} + \frac{0.02}{0.036\ W/m\text{-}K} + \frac{1}{566.7\ W/m^2\text{-}K}$$

$$R_u = 0.001765 + 0.000222 + 0.5556 + 0.033656\ m^2\text{-}K/W$$

$$\boxed{R_u = 0.5912\ m^2\text{-}K/W}$$

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Heat Flow Example Solution (3)

Now compute the total thermal resistance and the heat flow

$$R_T = \frac{R_u}{A} = \frac{0.5912\ m^2\text{-}K/W}{6\ m^2}$$

$$\boxed{R_T = 0.09854\ K/W}$$

Compute heat flow

$$T_o = 85°C \quad T_i = 45°C \qquad Q = \frac{T_o - T_i}{R_T}$$

$$Q = \frac{(85°C - 45°C)}{0.09854\ K/W} = \boxed{406\ W}$$

ANS

Note: No need to convert degrees C to K since we are taking the different of two Temperatures.

Convection Calculation Example

Example 7-2: Determine the thermal resistance for each of the following film conditions: a.) Natural convection in still air of vertical surface T_d = 20 C. b.) natural convection in still water where T_d = 30 C and T_w is 20 C
c.) forced convection of air with velocity of 4 m/s.

a.) Natural convection in still air of vertical surface

$T_d = 20\,°C$

$$h = 1.78\,T_d^{0.25}$$
$$h = 1.78(20)^{0.25}$$
$$h = 3.764 \ W/m^2\text{-}K$$
$$R = \frac{1}{h} = \boxed{0.266 \ K\text{-}m^2/W}$$

ANS

Example 7-2 Solution (2)

b.) Natural convection in still water $T_d = 30°C$ $T_w = 20\,°C$

$$h = 2.26(T_w + 34.3)(T_d)^{0.5}$$
$$h = 2.26(20 + 34.3)(30)^{0.5}$$
$$h = 2.26(54.3)(30)^{0.5}$$
$$h = 672.2 \ m^2\text{-}K/W$$

$$R = \frac{1}{h} = \frac{1}{672.2} = 0.00149 \ K\text{-}m^2/W$$
$$\boxed{R = 0.00149 \ K\text{-}m^2/W}$$

Example 7-2 Solution (3)

c.) forced convection of air with velocity of 4 m/s $V_{air} = 4\ m/s$

$$h = 4.54 + 4.1 V_{air}$$

$$h = 4.54 + 4.1(4\ m/s) = 20.94\ W/m^2\text{-}K$$

$$R = \frac{1}{h} = \frac{1}{20.94\ m^2\text{-}K/W} = \boxed{0.0478\ m^2\text{-}K/W}$$

Thermal Capacitance (Heat Capacity)

Thermal Capacitance - increase in heat required to make a unit change in temperature (SI unit J/K)

Heat Capacity (specific heat) - heat required to raise temperature of 1 kg of material by 1 K.

$$C_T = m \cdot S_T$$

Where:

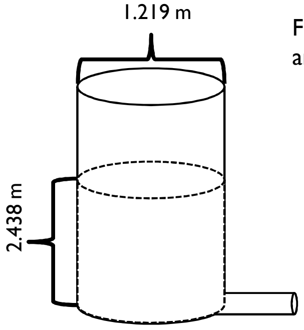
C_T = thermal capacitance

m = mass (kg)

S_T = heat capacity (J/kg)

Thermal Capacitance Example

Example 7-3: An insulated water storage tank shown below has a diameter of 1.219 m and is filled to a height of 2.438 m. Determine the thermal capacitance, C_T, of the water in the tank. Also determine the tank liquid capacitance, C_L.



Find the mass of water in the tank by using the tank volume and the density of water

$\rho = 1000\ \text{kg/m}^3$ $\quad h = 2.438\,\text{m} \quad d = 1.219\,\text{m}$

$S_T = 4190\ \text{J/kg - K}$

$$A = \frac{\pi \cdot d^2}{4} \qquad A = \frac{\pi \cdot (1.219\,\text{m})^2}{4} \qquad A = 1.167\text{m}^2$$

$$V_{tank} = h \cdot A \qquad V_{tank} = (2.438\,\text{m}) \cdot \left(1.167\,\text{m}^2\right) \qquad V_{tank} = 2.845\text{m}^3$$

$$\rho = 1000\frac{\text{kg}}{\text{m}^3} \qquad m_w = \rho \cdot V_{tank} \qquad m_w = 2845\text{kg}$$

Example 7-3 Solution (1)

Now find the thermal capacitance

$$S_T = 4190\frac{\text{J}}{\text{kg}\cdot\text{K}} \qquad m_w = 2845\text{kg} \qquad C_T = (2845\,\text{kg}) \cdot \left(4190\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$$

$$C_T = m_w \cdot S_T \qquad C_T = 1.192 \times 10^7\,\frac{\text{J}}{\text{K}}$$

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Compute the liquid capacitance

$$A = 1.167\text{m}^2 \qquad g = 9.807\frac{\text{m}}{\text{s}^2} \qquad \rho = 1000\frac{\text{kg}}{\text{m}^3}$$

$$C_L = \frac{A}{\rho \cdot g} \qquad C_L = 1.19 \times 10^{-4}\,\frac{\text{m}^3}{\text{Pa}}$$

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Characteristics of Mechanical Elements

Mechanical Resistance = Friction. Opposition to motion. Force required to increase velocity.

Types of Friction

Viscous Friction

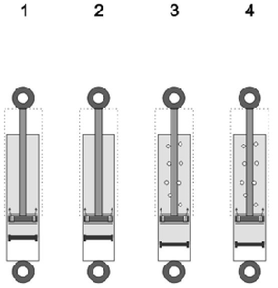
Friction of motion. Related to velocity.

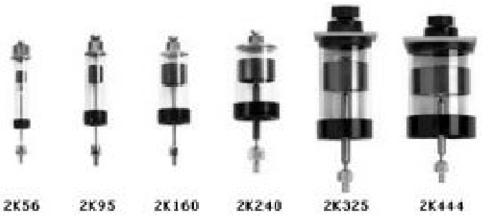
Coulomb Friction

Force independent of velocity

Characteristics of Mechanical Elements

Examples of viscous friction - shock absorber, dash pot





Linear viscous friction

$$R_m = B = \frac{F}{v} \quad (\mathrm{N\text{-}s/m})$$

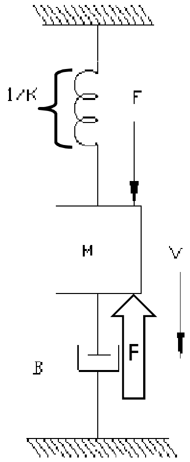
Non-linear viscous friction

$$R_m = B = \frac{\Delta F}{\Delta v}$$

$$R_m(v) = B(v) = \lim_{\Delta v \to 0} \frac{\Delta F}{\Delta v} = \frac{dF}{dv}$$

At operating point, use tangent line to find approximate B

Mechanical Energy Storage



Mechanical Capacitance - change in length of spring required to make unit increase in force. Inverse spring constant

$$C_m = \frac{1}{K} \ (\text{N/m})$$

Mechanical inertia - force required to make unit increase in acceleration. Proportional to mass.

$$F_{ave} = m \cdot \left[\frac{\Delta v}{\Delta t}\right] = m \cdot a_{ave}$$

$$F = \frac{dv}{dt} = m \cdot a$$

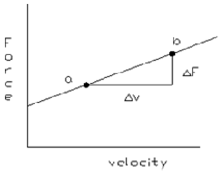
Mechanical Delay

Mechanical Dead-Time - time required to transport material from one place to another.

$$t_d = \frac{D}{v}$$

Where: D = distance material covers
v = velocity of material

Example 7-4: A mechanical system consists of a sliding load and a shock absorber. The force versus velocity curve is shown below. Find R_m and coulomb friction, F_c.



Run	Force=F (N)	Velocity=v (m/s)
a	7.1	10.5
b	9.6	15.75

Example 7-4 Solution

use $F = F_c + Bv$ $\quad B = R_m = \frac{\Delta F}{\Delta v}$

Use the point-slope form of the line

$$F - F_1 = \frac{\Delta F}{\Delta v}(v - v_1)$$

$F_1 = 7.1$ N $v_1 = 10.5$ m/s

$F - 7.1 = \frac{9.6 - 7.1}{15.75 - 10.5}(v - 10.5)$ $\quad F - 7.1 = 0.4762v - 5$

$F - 7.1 = \frac{2.5}{5.25}(v - 10.5)$ $\quad F = 0.4762v + 2.1$ N

$$\boxed{\begin{array}{l} B = 0.4762\ \text{N-s/m} \\ F_c = 2.1\ \text{N} \end{array}}$$

ANS

End Lesson 7: Thermal & Mechanical Models in Control Systems

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